数理解析研究所講究録
991 巻 1997 年 101-102

Tight Graphs and Their Primitive Idempotents*

Arlene A. Pascasio
De La Salle University
Manila, Philippines

March 4, 1997

Abstract

In this paper, we prove

Theorem 1. Let Γ denote a distance-regular graph with diameter $d \geq 3$. Suppose E and F are primitive idempotents of Γ, with cosine sequences $\sigma_0, \sigma_1, ..., \sigma_d$ and $\rho_0, \rho_1, ..., \rho_d$, respectively. Then the following are equivalent.

i) The entry-wise product $E \circ F$ is a scalar multiple of a primitive idempotent of Γ.

ii) There exists a real number ϵ such that

$$\sigma_i\rho_i - \sigma_{i-1}\rho_{i-1} = \epsilon(\sigma_{i-1}\rho_i - \sigma_i\rho_{i-1}) \qquad (1 \leq i \leq d).$$

Let Γ denote a distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \cdots > \theta_d$. In [1], Jurišić, Koolen and Terwilliger proved that the valency k and the intersection numbers a_1, b_1 satisfy

$$\left(\theta_1 + \frac{k}{a_1+1}\right)\left(\theta_d + \frac{k}{a_1+1}\right) \geq \frac{-ka_1b_1}{(a_1+1)^2}.$$

They called the graph *tight* whenever Γ is not bipartite, and equality holds above. Combining Theorem 1 with some of their results, we obtain

Corollary 2. Let Γ denote a nonbipartite distance-regular graph with diameter $d \geq 3$ and distinct eigenvalues $\theta_0 > \theta_1 > \cdots > \theta_d$. The following are equivalent.

i) There exist nontrivial primitive idempotents E, F of Γ such that (i), (ii) hold in Theorem 1.

ii) Γ is tight.

Moreover, if (i), (ii) hold then the eigenvalues of Γ associated with E, F are a permutation of θ_1, θ_d.

*This work was done when the author was an Honorary Fellow at the University of Wisconsin-Madison (September 1996 – September 1997) supported by the Department of Science and Techonology, Philippines.

Reference

[1] A. Jurišić, J. Koolen and P. Terwilliger, 1-Homogeneous Graphs (in preparation).

Acknowledgement

The author wishes to thank Professor Paul Terwilliger for his many valuable suggestions.